International Journal of Engineering Research and Reviews ISSN 2348-697X (Online)
Vol. 6, Issue 4, pp: (11-16), Month: October - December 2018, Available at: www.researchpublish.com

PID Controller Design for Time Delayed Unstable Systems Using CDM

[1]Mehmet ÇINAR

[1]Bitlis Eren University, Tatvan Vocational High School, Bitlis, TURKEY

Abstract: **The purpose of this work is to make use of the Coefficient Diagram Method (CDM) to design PID control systems with better performance in the control of time-delayed systems with first-order unstable transfer function. The PID controller designed with CDM and the standard PID controllers were designed and the results were compared. Designed with CDM, the PID controller has the advantages over the classic PID controller by providing the shortest settling time and the most robust behavior against variation in parameters.**

Keywords: **Coefficient Diagram Method, Time Delay, Unstable Systems.**

1. INTRODUCTION

In practice, there are time delays in the construction of many systems. This delay, also known as the dead time, is caused by the system itself, as it is caused by the fact that the system outputs can't be reused at the input or input-output signals can't be measured synchronously, and this causes a detrimental effect on the system's stability and transient characteristics. Despite the recent developments in control theory, PID controllers are still being used to control industrial time delay systems [2]. The most important reason for this is that the structure is simple and is generally successful in controlling many systems [3]. Furthermore, it is important to have practicality and durability for a wide working area [4]. There are various studies in the literature on the control of unstable systems. Morari and Zafiriou [5] derived formulas for calculating the PID parameters using the internal model control structure, Park et al. [6] proposed the PID-P structure, Poulin et al. [7] used the PID controller structure as the adaptive feature.

In this study, the Coefficient Diagram Method will be used to obtain better results against the problems encountered in the control of unstable systems. CDM was developed by Shunji Manabe in 1991 for the control of linear and time-invariant single-input single-output systems [8]. This work demonstrates that the CDM is an important tool for PID control of time delayed unstable systems because of the nature of the control system architecture and the advantages of the standard form that it uses. The most important features of the method are the use of a polynomial representation for the system and the controller, the use of a two-degree of freedom control system structure, the unit step response of the closed-loop system is usually not overburdened, the desired settling time is initially determined and designed, and the control system is robust against changes that may occur in the field [9].

2. COEFFICIENT DIAGRAM METHOD (CDM)

When designing the controller in the control systems, the controller must be selected at the lowest possible stable and minimum phase. If the controller does not have sufficient bandwidth and power constraints, the durability feature will be weak, although stability requirements may be met. CDM (Coefficient Diagram Method) has been developed by Manabe to get over these problems. Method disruptive effects have advantages over other methods in terms of adequate disintegration, durability and cost. Because of the simplicity of the method design process, good control systems can be designed without much difficulty. In addition to providing information on stability and step response such as Bode and Nyquist diagrams, CDM is also very successful in questioning the stability, time response and endurance characteristics that are important for high-grade characteristic polynomial systems. In particular, it is a great advantage that the control system exhibits resistance to system parameters and, if necessary, the system itself is resistant to limited uncertainties.

A. CDM Control System Structure

The CDM block diagram for a single-input single-output system is shown in Figure 1. *r* is the reference input, *y* is the output, *u* is the control signal, and *d* is the disturbing signal that affects the system. *N (s)* is the polynomial for the transfer function and *D(s)* is the polynomial for the denominator. The *A(s)* denominator polynomial, the *F(s)* reference fraction polynomial and the *B(s)* feedback fraction polynomial are given for the controller transfer function. In this structure, unstable pole-zero erasures are eliminated and fewer integrator elements are used in practice.

Closed loop system output statement:

$$y = \frac{N(s)F(s)}{P(s)} r + \frac{A(s)N(s)}{P(s)} d \tag{1}$$

If it is a *P(s)* characteristic polynomial;

$$P(s) = D(s)A(s) + N(s)B(s) = \sum_{i=0}^{n} a_i s^i \tag{2}$$

is expressed. The A (s) and B (s) polynomials are expressed as follows.

$$A(s) = \sum_{i=0}^{p} l_i s^i \quad \text{and} \quad B(s) = \sum_{i=0}^{q} k_i s^i \tag{3}$$

The rank m of the N (s) polynomial and the rank n of the polynomial D (s) must be $m \le n$.

In CDM, the design parameters τ, stability index γ_i and stability limit index γ_i' are in the form of characteristic polynomial coefficients;

$$\gamma_i = \frac{a_i^2}{a_{i+1} a_{i-1}} \tag{4.a}$$

$$\tau = \frac{a_1}{a_0} \tag{4.b}$$

$$\gamma_i' = \frac{1}{\gamma_{i-1}} + \frac{1}{\gamma_{i+1}} \tag{4.c}$$

It is defined as. The following stability indices are used for the characteristic polynomial in CDM.

$$\gamma_1 = 2{,}5 \quad \gamma_i = 2 \ \text{ i=2 ~ (n-1)}, \quad \gamma_0 = \gamma_1 = \infty \tag{4.d}$$

The most important features of CDM are as follows:

a) The settlement period for the unit step function response of the closed loop system is around 2.5τ, which is smaller than other methods.

b) The step function response of the designed control system is indefinite.

c) For the same τ and zero order polynomial, the unit step response of the standard structure is independent of the characteristic polynomial order and remains the same.

d) The gain and phase boundaries of the controller designed with CDM are obtained at the desired optimum values.

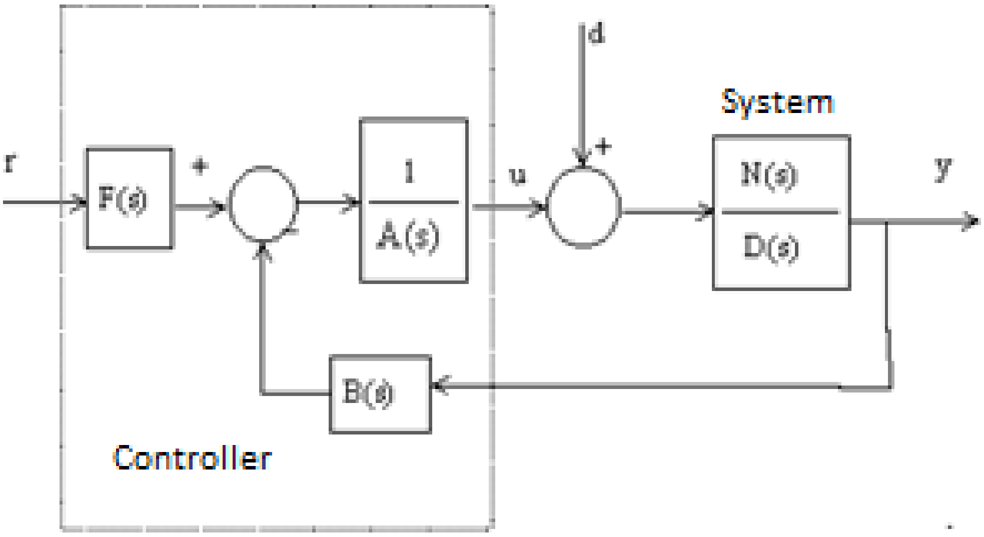

Figure 1. Block Diagram of the CDM Control System

3. DESIGN PROCEDURE

First-order unstable time-delayed systems commonly used in the industry are shown as follows.

$$G(s)=\frac{K}{Ts-1}e^{-\theta s} \tag{5}$$

As can be seen from the structure of the system, the open loop unit step function of the system is unlimited in response to the pole in the right half plane. A general and systematic design procedure for PID control with the aid of CDM for the unstable systems given in Equation (5) can be given as follows:

A. Information Specified Before the Start of the Design:

A.1. Approximate Equivalent Use for Time Delay:

For $e^{-\theta s}$ expressing time delay

$$e^{-\theta s}\approx\frac{-\frac{\theta}{2}s+1}{\frac{\theta}{2}s+1} \tag{6}$$

Pade approach expressed as the system structure in Equation 5 is obtained as follows.

$$G(s)=\frac{\frac{-K\theta}{2}s+K}{\frac{T\theta}{2}s^2+\left(T-\frac{\theta}{2}\right)s-1} \tag{7}$$

linear-time-invariant equivalent is obtained.

A.2. Selecting Controller Polynomials *A(s)* , *B(s)* and *F(s)*:

$$G(s)=\frac{n_1s+n_0}{d_2s^2+d_1s+d_0}=\frac{N(s)}{D(s)} \tag{8}$$

The coefficients of Equation (8) are abbreviated as follows.

$$d_2=\frac{T\theta}{2},\quad d_1=T-\frac{\theta}{2},\quad d_0=-1,\quad n_1=-\frac{K\theta}{2},\quad n_0=K \tag{9}$$

$$N(s) = n_1 s + n_0 \tag{10.1}$$

$$D(s) = d_2 s^2 + d_1 s + d_0 \tag{10.2}$$

$$A(s) = l_1 s \tag{10.3}$$

$$B(s) = k_2 s^2 + k_1 s + k_0 \tag{10.4}$$

$$P_{target}(s) = A(s)D(s) + N(s)B(s) \tag{10.5}$$

$$P_{target}(s) = l_1 s\ .(d_2 s^2 + d_1 s + d_0) + (n_1 s + n_0).(k_2 s^2 + k_1 s + k_0) \tag{10.6}$$

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right) \tag{10.7}$$

are obtained.

A.3. Selection of the Equivalent Time Constant and Stability Index as Key Parameters for Design

If the system is in the nth order, the target polynomial is selected as follows.

$$P_{target} = \frac{\tau^n s^n}{\gamma_1^{n-1} \gamma_2^{n-2} \ldots \gamma_{n-1}^1} + \frac{\tau^{n-1} s^{n-1}}{\gamma_1^{n-2} \gamma_2^{n-3} \ldots \gamma_{n-1}^1} + \ldots \frac{\tau^3 s^3}{\gamma_1^2 \gamma_2} + \frac{\tau^2 s^2}{\gamma_1} + \tau s + 1 \tag{11}$$

When calculations are made $\gamma_1 = 2.5$, $\gamma_{2,3,\ldots n-1} = 2$ are taken. When designing by coefficient diagram method *t* (settlement time) = (2.5 ~ 3) τ is taken.

B. Calculation of Characteristic Polynomial and Controller Polynomial Coefficients During Design:

If the product of the coefficients of P_{target} is done, the following equations are obtained.

$$l_1 d_2 + n_1 k_2 = \frac{\tau^3}{\gamma_1^2 \gamma_2} \tag{12.1}$$

$$\theta = \frac{2\tau^3}{\gamma_1^2 \gamma_2 (l_1 T - k_2 K)} \tag{12.2}$$

$$l_1 d_1 + n_1 k_1 + n_0 k_2 = \frac{\tau^2}{\gamma_1} \tag{12.3}$$

$$l_1 d_0 + n_1 k + n_0 k1 = \tau \tag{12.4}$$

$$n_0 k_0 = 1 \tag{12.5}$$

Equations are obtained. The coefficients of the transfer function in the coefficient diagram method are equalized to the PID controller coefficients and the following equations are obtained.

$$C(s) = \frac{B(s)}{A(s)} \tag{13}$$

$$\frac{k_2 s^2}{l_1 s} + \frac{k_1 s}{l_1 s} + \frac{k_0}{l_1 s} = K_p + \frac{K_p}{T_i s} + K_p T_d s \tag{14}$$

$$K_p = \frac{k_1}{l_1} \ , \quad T_d = \frac{k_2}{k_1} \ , \quad T_i = \frac{k_1}{k_0} \tag{15}$$

Equations are obtained.

C. Controls after Design:

When designing with coefficient diagram method, *N(s)* and *D(s)* polynomials are obtained by calculating target polynomial coefficients. The graph of the system is drawn with the help of these two polynomials. To demonstrate the efficiency of the given design process, a design application will be implemented in the following section.

4. DESIGN APPLICATION

The transfer function of the first order time lag system is given in equation 16.

$$G(s) = \frac{e^{-1,2s}}{(1,5s-1)} \qquad (16)$$

A disturbance signal is applied to the system as a step function with t = 40 s and amplitude is 0.1. Accordingly, it is aimed to control the system in the best possible way considering the time response characteristics.

This system was previously described by Park et al. When controlled by the PID-P control system proposed by [6], the PID-P controller parameters are determined as $K_p = 0.0672$, $T_i = 1.4016$, $K_f = 1.118$ and $T_d = 4.8001$. Here, taking into account the control system performance obtained by this method, a CDM based PID controller will be designed and the result of this design will be compared to the performance of the two control systems in which the performance of the control system is derived.

The CDM design procedure has been implemented as follows, taking into consideration the time response characteristics of the two controllers given above (especially the duration of the settlement and the amplitude of the control signal). The response to the unit step function of the control system is given in Figure (2) with the time responses of the control system designed with two different methods.

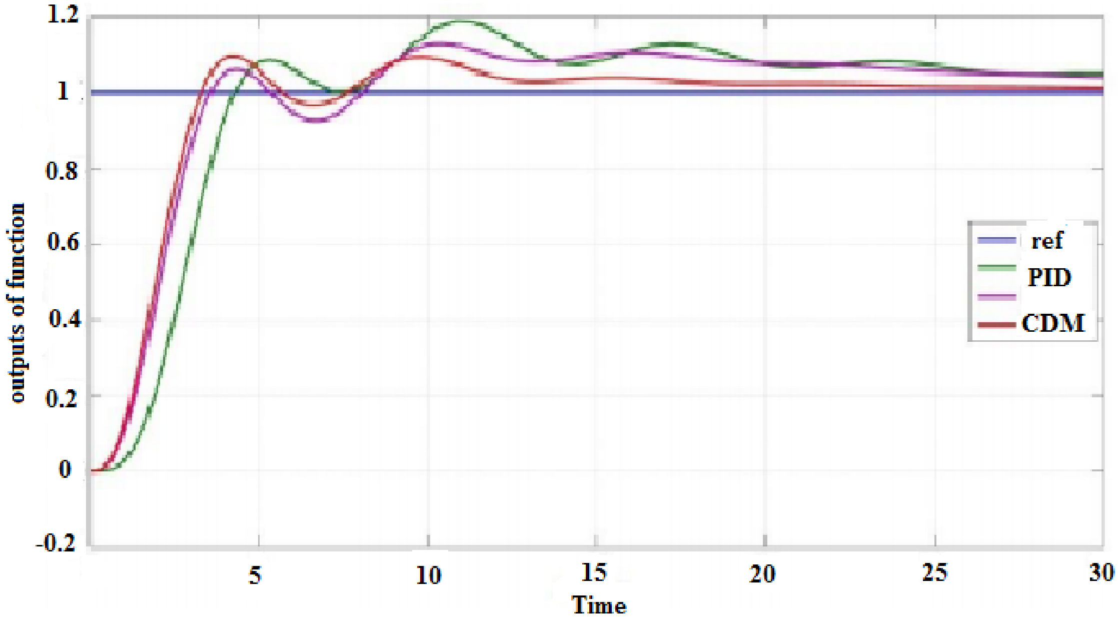

Figure 2. Unit Step Response

5. CONCLUSION

Considering the time responses of the controller and the closed-loop system obtained for the two separate methods, it is seen that the CDM-based PID control system becomes stable in a shorter time than the conventional PID control system. The coefficient diagram method (CDM) produces a stable solution by ensuring that the system becomes stable in a shorter time than other methods.

REFERENCES

[1] Gorecki, H., Fuksa, S., Grabowski, P and Korytowski, A., Analysis and Synthesis of Time Delay Systems. John Wiley, NY, 1989.

[2] S. Majhi and D.P. Atherton, “Autotuning and controller design for processes with small time delays”, IEE Proc.-Control Theory Appl., vol. 146, no. 5, pp.415-425, 1999.

[3] W. Tan, J. Liu and P.K.S. Tam, “PID tuning based on loop-shaping H-inf control”, IEE Proc.-Control Theory Appl., vol. 145, no. 6, pp.485-490, 1998.

[4] K.J. Astrom and T. Hagglund, “Automatic tuning of simple regulators with specificatins on phase and amplitude margins,” Automatica, vol. 20, no. 5, pp. 645–651, 1984.

[5] M. Morari and E. Zafiriou, Robust Process Control, Prentice Hall, Englewood, NJ, 1989.

[6] J. H. Park, S.W. Sung and I. Lee, “An enhanced PID control strategy for unstable proceses”, Automatica, vol.34(6), pp.751-756, 1998.

[7] E. Poulin, A. Pomerleau, A. Desbiens and D. Hodouin, "Development and evaluation of an auto-tuning and adaptive PID controller“, Automatica, vol.32, pp.71-82, 1996.

[8] S. Manabe, “Unified interpretation of classical, optimal and $H\infty$ control”, Journal of SICE, vol.30, no.10, pp.941-946, 1991.

[9] S. Manabe, "Coefficient Diagram Method", 14^{th} IFAC Symposium on Automatic Control in Aerospace, Seoul, 1998